NOTE

Colours in Cellophane

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If transparent cellophane tape is viewed between crossed polarizers using a white light source, the emergent light is coloured. The reason it shows this property is that during its manufacture cellophane tape is stretched so that its organic molecules tend to straighten and align in the direction of stretch. This results in the dielectric constant and hence the refractive index being different in the direction along the tape from that in the perpendicular direction. These two directions act as principal axes and the polarization components parallel to and perpendicular to the length of the tape travel with different velocities. The principal axis with the larger of the two refractive indices (n_s) is known as the slow axis, while the other is the fast axis (n_f).

When plane-polarized light is incident on cellophane tape with the plane of polarization at 45° to the axes of the tape it can be thought of as two components of equal intensity traveling along the fast and slow axes which start off in phase. On emergence one vibration will be retarded with respect to the other. If the tape is of thickness t then the retardation of the cellophane is defined as

$$R = (n_s - n_f)t.$$

The difference $n_s - n_f$ is known as the birefringence. The phase difference between the two emergent vibrations is

$$\delta = 2\pi(n_s - n_f)t/\lambda = 2\pi R/\lambda.$$

This phase difference is wavelength dependent so that the state of polarization of the emergent light is different for each wavelength present. In general, the light is elliptically polarized but the shape of the ellipse varies from circular for some wavelengths ($\delta = \pi/2, 3\pi/2$, etc.) to plane for other wavelengths ($\delta = \pi, 2\pi$, etc.).

If the second polarizer has its transmission axis at 90° to the first, then light of the wavelength for which the thickness t is such that $\delta = 2\pi, 4\pi$, etc., will emerge plane polarized in the same direction as the light incident on the cellophane. This wavelength will be cut out by the second polarizer and as one moves away from this cutoff wavelength the transmission will increase until it becomes a maximum at $\delta = \pi, 3\pi$, etc., when the emergent beam is plane polarized in the direction of the transmission axis of the second polarizer. This variation of transmission with wavelength means that the emergent light is coloured.

When the thickness of the cellophane used is varied the wavelengths of maximum and minimum transmission are altered so that the colour of the emergent light changes. Large changes of colour are observed for relatively thin layers of cellophane but for thick layers of cellophane a number of cutoff wavelengths are obtained across the spectrum. The emergent light then appears white but if the spectrum is viewed in a spectrometer a number of dark bands are observed centred around the cutoff wavelengths.

Samples were made containing 1–15 layers of cellophane held between crossed sheets of "Polaroid." The direction of stress of the cellophane was fixed at 45° to the axes of both polarizers. The samples were placed in turn in a Unicam SP800 scanning spectrophotometer and the variation of absorbance with wavelength was obtained over the wavelength range 400–700 nm. Two of these curves are illustrated in Fig. 1.

The CIE tristimulus values were calculated from the absorbance curves and the corresponding chromaticity coordinates were plotted in Fig. 2. As can be seen from the tabulated results in Table I, with the thickness of cellophane used in these measurements, 3.35×10^{-2} mm, large visual colour changes were obtained for 1–6 layers, while for greater thicknesses little change is evident to the eye, all appearing white of very slightly different hue.

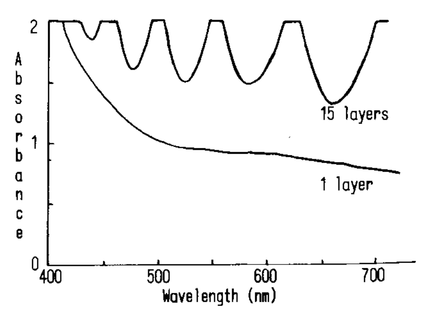

FIG. 1. Plot of absorbance against wavelength for 1 and 15 layers of cellophane.

TABLE I. CIE colorimetric data for cellophane colours (Ill. *C* and 1931 standard observer).

Number of layers	Colour	*x*	*y*	*Y*	Dominant wavelength (nm)	Excitation purity (%)
1	Pale yellow	0.381	0.408	11.805	574	43.9
2	Strong blue	0.188	0.264	4.445	486	49.6
3	Magenta	0.451	0.319	4.554	614	38.5
4	Lime green	0.288	0.528	6.639	546	52.5
5	Purple	0.326	0.240	3.037	538	33.9
6	Pale green	0.341	0.374	5.277	569	23.9
7	White	0.316	0.402	3.420	555	23.6
8	White	0.381	0.363	3.660	583	30.9
9	White	0.313	0.392	3.451	554	21.3
10	White	0.368	0.357	2.959	582	27.1
11	White	0.331	0.386	2.889	563	27.1
12	White	0.357	0.371	2.608	575	27.3
13	White	0.342	0.383	2.280	568	27.1
14	White	0.356	0.385	2.301	569	27.9
15	White	0.349	0.377	2.001	572	26.9

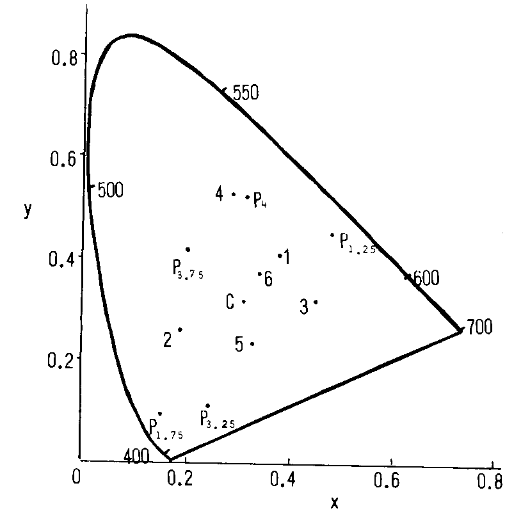

FIG. 2. Chromaticity coordinates of colours produced by 1–6 layers of cellophane. Most saturated predicted colours for intermediate layers are shown by points P1.25, etc.

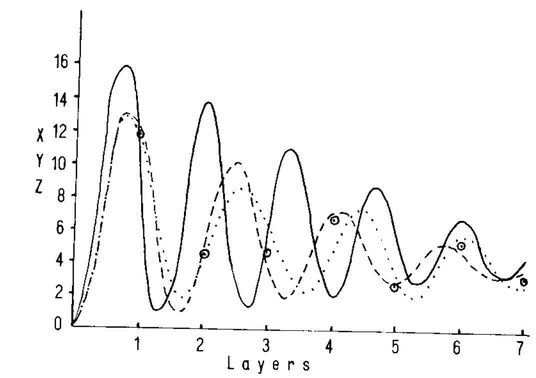

FIG. 3. Predicted values of *X* (· · ·), *Y* (- - -), and *Z* (—) for various thicknesses of cellophane. Experimental values of *Y* are plotted as circled points.

The graphs for the thicker layers (10–15) were used to determine the retardance of the cellophane. This was done by determining the wavelengths of minimum absorbance (λ) and plotting these against the natural integers when the gradient is $1/R$. The theory of this method is given in a paper by S. D. Cloud[1] and the result obtained for one layer of cellophane was 3.38×10^{-7} m, leading to a value for the birefringence of 0.0100 ± 0.0004.

Along with the changes in colour which occur it is obvious to the eye that the total amount of light transmitted by each sample does not decrease steadily with increasing thickness; this is evident from the values of Y in Table I. Cloud's paper derives an expression for the intensity transmitted at each wavelength:

$$I = \frac{1}{4} T_p{}^2 T_c (1 - \cos 2\pi R/\lambda),$$

where T_p is the transmittance of the polarizers and T_c the transmittance of the cellophane. This expression was used to predict the values of Y by weighting it with the relative luminous efficiency values $V(\lambda)$ and integrating the result over the wavelength range 400–700 nm:

$$Y = \frac{1}{4} \int_{400}^{700} T_p{}^2 \, T_c V(\lambda)(1 - \cos 2\pi R/\lambda) d\lambda.$$

The theoretical variation of the resulting values of Y is shown in Fig. 3 and the corresponding experimental values are indicated; the agreement is good. Similar calculations for X and Z using the $\bar{x}(\lambda)$ and $\bar{z}(\lambda)$ curves as weighting functions allow prediction of the colours produced by various thicknesses (Fig. 2) and indicates that many more saturated colours would have been observed if intermediate thicknesses of cellophane had been available.

Acknowledgements

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1. S. D. Cloud, Birefringence experiments for the introductory physics course, *Am. J. Phys.* **41**, 1184–1188 (1973).